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How Much Do I Know About Mathematical Modeling?

Ji-Won Son
SUNY University at Buffalo

Hyunyi Jung
Marquette University

Ji Yeong I
Iowa State University, jiyeongi@iastate.edu

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Abstract

Despite the importance of teachers' conception of mathematical modeling, limited attention is given to this area in the current literature. In this study we examined 78 preservice teachers' (PSTs) views of mathematical modeling and how their conceptions are reflected in their performance of mathematical modeling problems. Analyses of survey responses revealed that our PSTs seem to develop narrow views of mathematical modeling. In addition, although a large portion of PSTs mistook mathematical modeling with mathematical models or with traditional word problems, we found a positive association between PSTs' conceptions of mathematical modeling and their mathematical modeling abilities.

Keywords

Modeling, Teacher Education-Preservice

Disciplines

Analysis | Discrete Mathematics and Combinatorics | Higher Education | Theory and Algorithms

Comments

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HOW MUCH DO I KNOW ABOUT MATHEMATICAL MODELING?

Ji-Won Son
University at Buffalo-SUNY
jiwonson@buffalo.edu

Hyunyi Jung
Marquette University
hyunyi.jung@marquette.edu

Ji Yeong I
Iowa State University
jiyeongi@iastate.edu

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Keywords: Modeling, Teacher Education-Preservice

Introduction

Mathematics education community at large has recognized the importance of mathematical modeling at school level, which concerns how well students are prepared to solve real-world problems that they encounter beyond school, to solve problems in their future professions, as citizens and in further learning (Galbraith & Stillman, 2006). The Common Core State Standards for Mathematics (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) also highlights mathematical modeling as one of the eight Standards for Mathematical Practice for all grades but also as conceptual category in high school. Thus, preparing effective teachers of mathematics who promote students' conceptual understanding and mathematical modeling abilities is one of the most urgent problems facing teacher educators. The purpose of this study is to explore PSTs' conceptions of mathematical modeling and effective modeling instruction and to investigate any relationship that might exist among PSTs' conceptions of effective modeling instruction, mathematical modeling, and their mathematical modeling performance. In exploring the relationship between PSTs' conception of effective mathematical modeling instruction and their mathematical modeling abilities, we specifically focus on two popular modeling problems – (a) Deciding a departing time for airport and (b) finding the best estimate of the total number of people in concert. The research questions that guided this study are: (a) What are the characteristics of PSTs' thinking about effective mathematical modeling instruction and mathematical modeling? and (b) Is there any relationship among PSTs' conceptions of effective mathematical modeling instruction, mathematical modeling, and their mathematical modeling performance?

Theoretical Perspectives

Mathematical modeling is a powerful vehicle for students' mathematical learning. However, the term "mathematical modeling" is easily confused. Research reported several confusions teachers and students have. For example, the term of mathematical modeling is often considered as mathematical models or traditional word problems. Although there exist distinct differences between "modeling as content" and "modeling as vehicle" (Galbraith & Stillman, 2006), teachers tend to "treat [mathematical modeling] more as a venue for learning other mathematics" (Zbiek & Conner, 2006, p. 89). Mathematical modeling involves a cyclical process as shown in Figure 1 in which real-life problems are understood and translated into mathematical language (*formulate*), solved within a symbolic system (*compute*), and the solutions tested back within the real-life system (*interpret, validate, and report*).

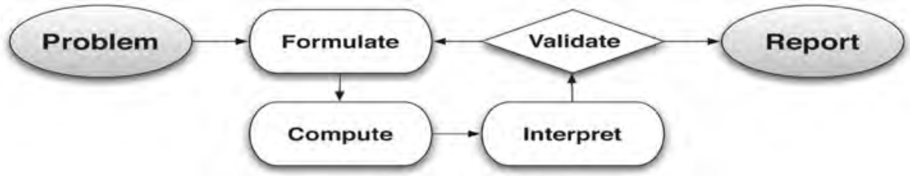

Figure 1. The basic modeling cycle introduced in the CCSSM (NGA & CCSSI, p. 72).

In this study, using the three meanings of mathematical modeling by Stanic and Kilpatrick (1989) and the three teaching approaches to mathematical modeling by Schroeder and Lester (1989), we explored PSTs' conceptions of mathematical modeling and effective mathematical modeling instruction. Building on Schroeder and Lester's framework, the following three perspectives of mathematical modeling instruction can be identified in mathematics classrooms: (1) teaching *for* mathematical modeling, (2) teaching *about* mathematical modeling, and (3) teaching *through* mathematical modeling. In addition, drawn from Stanic and Kilpatrick, we believe that *mathematical modeling as art* should be a goal of effective mathematical modeling instruction. According to them, three different meanings were attributed to the notion of mathematical modeling in mathematics education-- *mathematical modeling as means to a focused end (content), mathematical modeling as a skill,* and *mathematical modeling as art.* In the first perspective, mathematical modeling can be viewed just as *content* to practice skills. Similarly, the second perspective considers mathematical modeling as one skill taught in school mathematics. In contrast, in the third perspective, mathematical modeling should be viewed as an act of discovery through creative use of mathematical thinking.

Methods

78 PSTs from two different university sites were invited for this study. Participants had some experience of solving modeling problems within their class work. In the beginning of the semester, all participants completed the tasks shown in Figure 2. They showed misconceptions on mathematical modeling and did not provide a realistic answer to the problems. Two 3-hour sessions were devoted to help them understand mathematical modeling. By the end of the semester, a written task (see Fig. 2), was used for the study as part of final assessment.

Part 1: Please answer the following questions in as much detail as possible.

1. When people say mathematical modeling, what does the word "mathematical modeling" mean to you?
2. What do you believe constitutes effective mathematical modeling instruction?

Part 2: Solve the following problems.

1. Your best friend is coming to visit. She told you that her bus arrives at 4:00 pm. You live 10 miles away from the bus station. The speed limit is 40 miles per hour. When should you leave your house? Explain your reasoning.
2. A popular band recently came to a music festival. A field of size 100 m by 200 m was reserved for the audience. The concert was completely sold out and the field was packed with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people that attended the concert? 10,000, 20,000, 100,000, 200,000, or 400,000? Explain your reasoning.

Figure 2. Main task of this study.

For the analysis of PSTs' written response to mathematical modeling and effective mathematical modeling instruction, we used an inductive content analysis approach. PSTs' responses to the notions of mathematical modeling and effective mathematical modeling instruction were categorized based

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

on themes emerging as researchers read multiple cases. Then we explored the subcategories under each analytical aspect according to the framework (see Table 1 later). For the modeling task, we first created a rubric based on correctness of PSTs' responses to each item and then assigned a score for each item. To examine relationship among PSTs' conceptions of mathematical modeling, effective mathematical modeling instruction, and their mathematical modeling performance, we run SPSS statistical program (e.g., ANOVAs).

Summary of Selected Findings

Psts' Conceptions of Effective Mathematical Modeling Instruction

To investigate PSTs' conception of effective mathematical modeling (MM) instruction, we reviewed their responses and classified the responses into four aspects based on common themes (see Table 1). Out of the four aspects, the most popular category is *teaching* aspect (i.e., what instructional strategies or teaching practice need for effective MM instruction?), followed by mathematical modeling *steps* aspect (i.e., what step is required for MM lesson?), *problem features* aspect (i.e., what is considered as a good problem for MM instruction?), and *purpose* aspect (i.e., what is a good MM lesson aimed at?).

Table 1: Four Aspects of PSTs' Conception of a Good Mathematical Modeling Lesson and Frequencies

Category	Sub-category	# of PSTs	Relation to 3 MM approaches
1. Purpose aspect (29)	a. To find a realistic solution	10	For
	b. To develop critical/ logical /reflective thinking	14	Through
	c. To develop a good understanding of math	5	Through
2. Problem aspect (39)	a. Real-life problems	14	Through
	b. Problems that requires student prior knowledge	5	Through
	c. Practice problems that use the same technique	4	For
	d. Problems that require multiple solutions	11	Through
	e. Problems that require explanations	5	Through
3. Mathematical modeling steps aspects (42)	a. Structuring a lesson for all five modeling steps	14	About
	b. Identifying problem	12	About
	c. Devising a strategy	3	About
	d. Carrying out	2	About
	e. Interpreting	11	About
	f. Validating	5	About
4. Teaching aspect (78)	a. Emphasizing different ways of solving problems	20	Through
	b. Giving examples about how to solve	10	For
	c. Giving definitions on mathematical modeling	9	For
	d. Giving enough time to work on problems	6	For/Through
	e. Providing a direct, clear direction and structure	25	For/About
	g. Lessons that are interesting to students	8	Through

Note. Majority of PSTs addressed multiple categories. These responses were coded in multiple categories as long as the categories were present in their written responses.

After identifying the four aspects, we collectively considered them to categorize PSTs' conceptions of effective MM instruction into the three groups by referring to Schroeder and Lester's (1989) identification. Out of 78 participants, 42 participants considered effective MM instruction as teaching *about* mathematical modeling, 13 participants as teaching *through* mathematical modeling, and 20 participants as teaching *for* mathematical modeling. This finding indicates that despite the consistent emphasis on teaching *through* mathematical modeling in current mathematics education, a large portion of our PSTs still did not have a clear view of teaching *through* mathematical modeling. In a similar way, we categorized PSTs' conception of MM into three groups by referring to Stanic and Kilpatrick's (1989) identification. Out of 78 PSTs, 42 PSTs considered MM as *content*, 22 PSTs as a *skill*, and 14 PSTs as *art* of discovery.

Relationship between PSTs' Conceptions and Their Modeling Performance

For the first problem that asks students to decide when to leave their house, about 42 % PSTs responded that they would leave their house at times before 3:45 pm to go pick up their friend (correct realistic answer); 58% responded that they would leave their house at 3:45 pm. For the second modeling problem that requires students to determine the best estimate of the total number of people that can attend the concert in the size 100 m by 200 m, 42 % PSTs responded that there were 100,000 fans (correct realistic answer) whereas 38% provided a mathematically correct answer, 20,000. After coding PSTs' written responses and grading mathematics tasks, we quantified the data analysis result to examine relationship among PSTs' conception of mathematical modeling, a good mathematical modeling lesson, and their mathematical modeling performance. A chi-squared test showed that there is a positive relationship between PSTs' conception of mathematical modeling and their conception of effective mathematical modeling instruction, $\chi^2 = 16.888$, $df = 2$, $p = 0.002$**.** In addition, there was a significant difference of mean scores concerning mathematical modeling competence among groups of PSTs who perceived different views on mathematical modeling, $F(2, 73) = 3.292$, $p = .024$. PSTs who perceived mathematical modeling as *art* showed highest mean scores in the mathematical modeling tasks, followed by PSTs who with mathematical modeling as *means* to a focused end.

Discussion and Implications

This study contributes to the current literature on mathematical modeling and the knowledge base of teacher education. The findings of this study suggest that teacher educators need to find a better way to help PSTs perceive mathematical modeling as *art* and effective mathematical modeling instruction as teaching mathematics *through* mathematical modeling (Son, 2016). One approach would be: Have PSTs experience three different perspectives of teaching mathematical modeling and compare affordances and limitations of each approach. Then teacher educators need to give PSTs more opportunities to experience teaching *through* mathematical modeling in their mathematics methods courses where PSTs engage in mathematical modes of thought by analyzing and interpreting the problems. Furthermore, intervention studies that experiment with these suggestions are needed to find a better way to support PSTs' conceptions regarding mathematical modeling, modeling lessons and their mathematical modeling abilities.

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